Control Lec 7

quiz

$$GH(s) = \frac{1}{s^2(s+1)}$$

Draw the Polar Plot & determine PM & GM and stability.

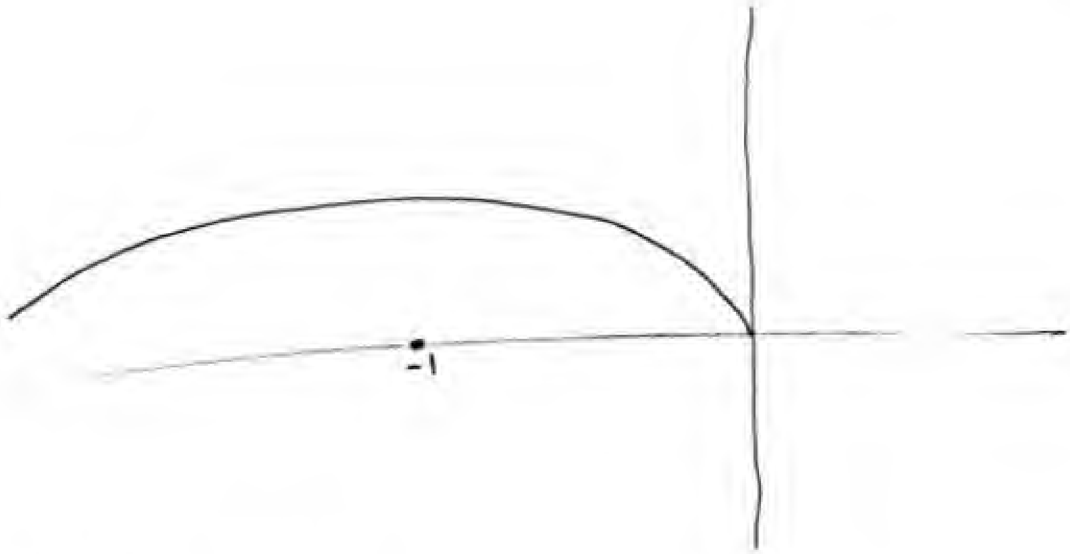
Sol

$$GH(J\omega) = \frac{1}{(J\omega)^2\,(J\omega+1)}$$

$$|GH(J\omega)| = \frac{1}{\omega^2\sqrt{\omega^2+1}}$$

$$\angle GH(J\omega) = -180 - \tan^{-1}(\omega)$$

ω	0	0.25	0.5	0.75	1	1.5	∞
$\lvert\ \rvert$	∞	13.52	3.58	1.42	0.707	0.24	0
$\angle$	-180	-194°	-206.5	-216.5	-225°	-236.3°	-270



→ system unstable

$GM = \frac{1}{\infty} = 0 < 1$ → unstable

$\omega = 0.85 \Rightarrow |GH| = 1.054$

$\therefore \omega_{gc} = 0.85$

$PM = -180 + \phi(\omega_{gc})$

$= -40.36^{\circ}$ (-ve) unstable

$\boxed{\text{Notes}}$

start point $\Rightarrow \omega = 0$ → Real $\angle 0$ ⇒ type 0

→ $\infty \angle -90$ ⇒ type 1

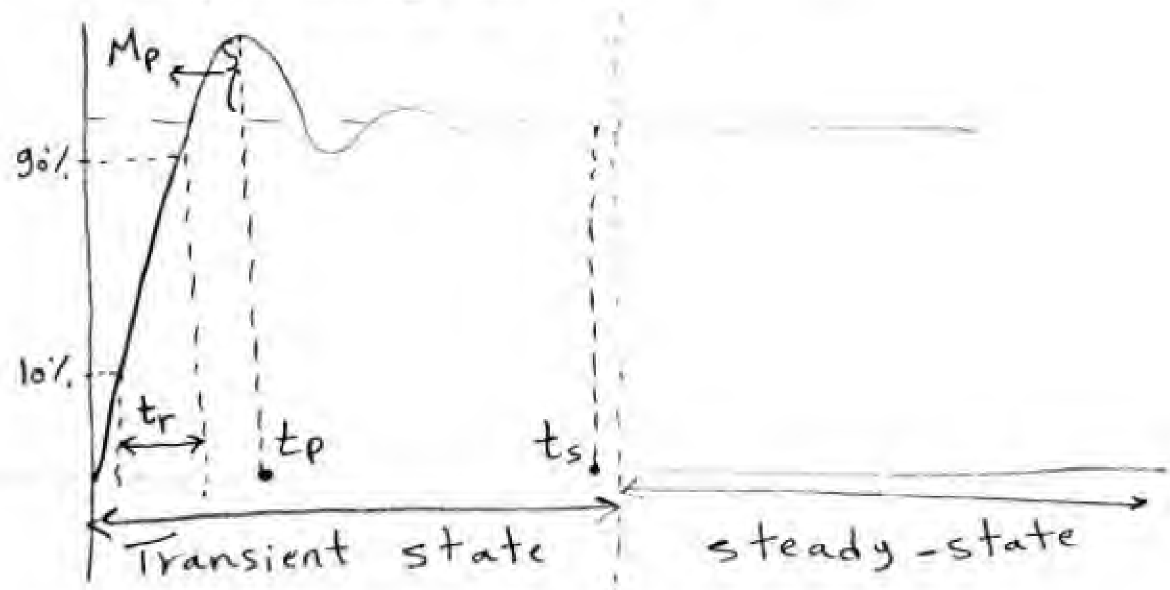
→ $\infty \angle -180$ ⇒ type 2

end point $\Rightarrow \omega = \infty$ → $0 \angle (n_p - n_z) \times (-90^{\circ})$ for any type

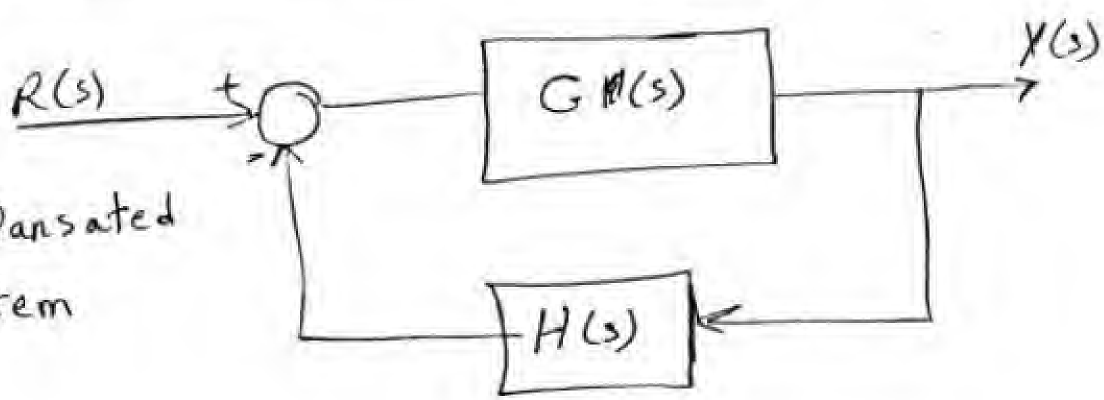
* Design of Controllers

The Controllers are used to improve the system performance

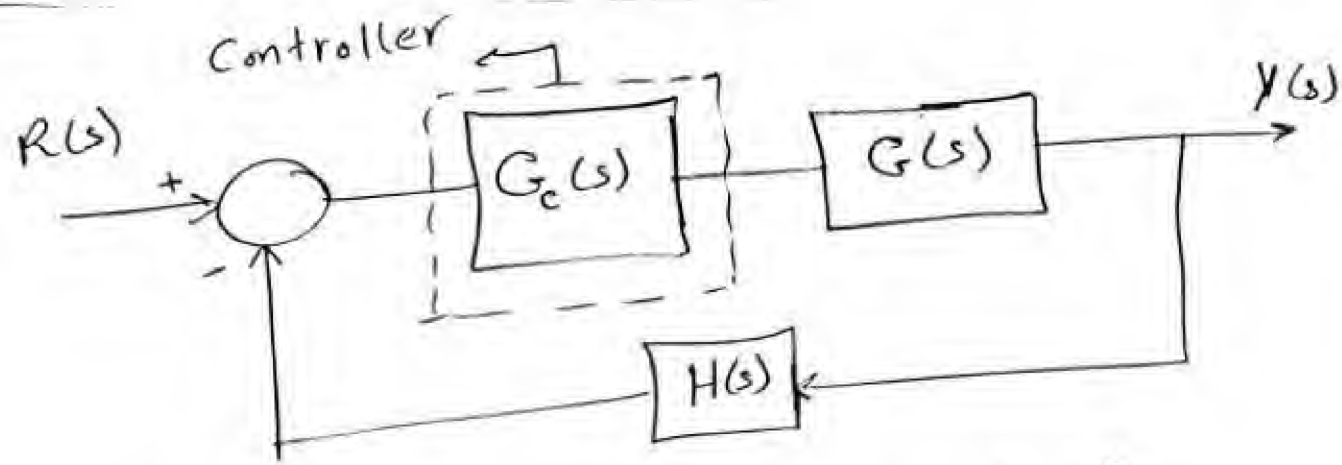
- → system dynamic (overshoot, t_r, t_s...)
- → steady-state error.



~~→ Common structures Controllers~~



un Compansated system

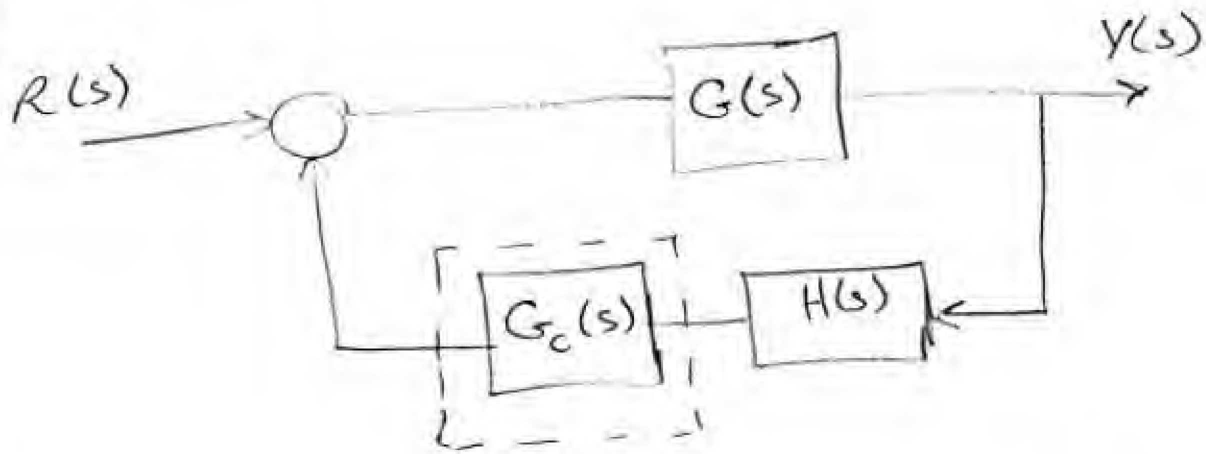


"Compansated system"

Common structures of controllers (Compansated)

1 Compansated system.

2



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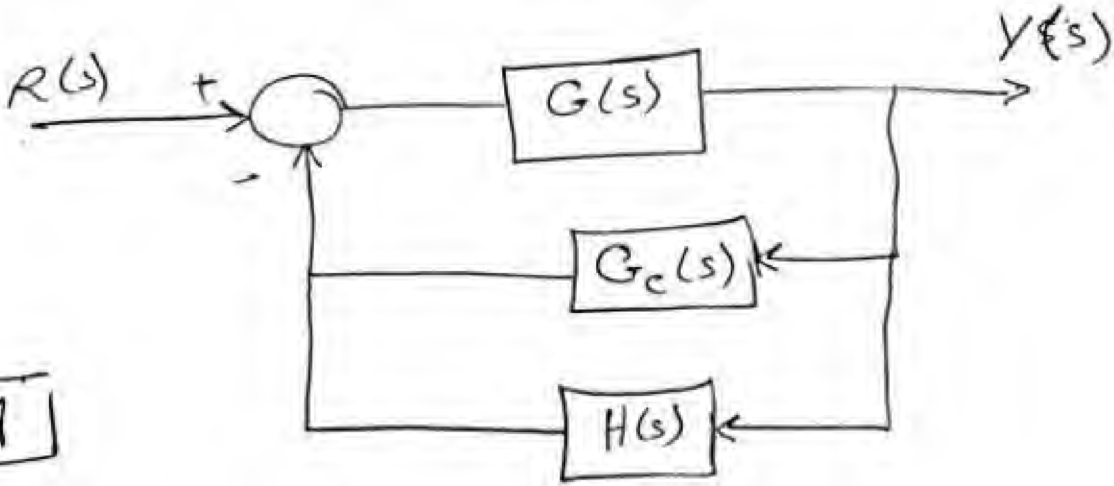

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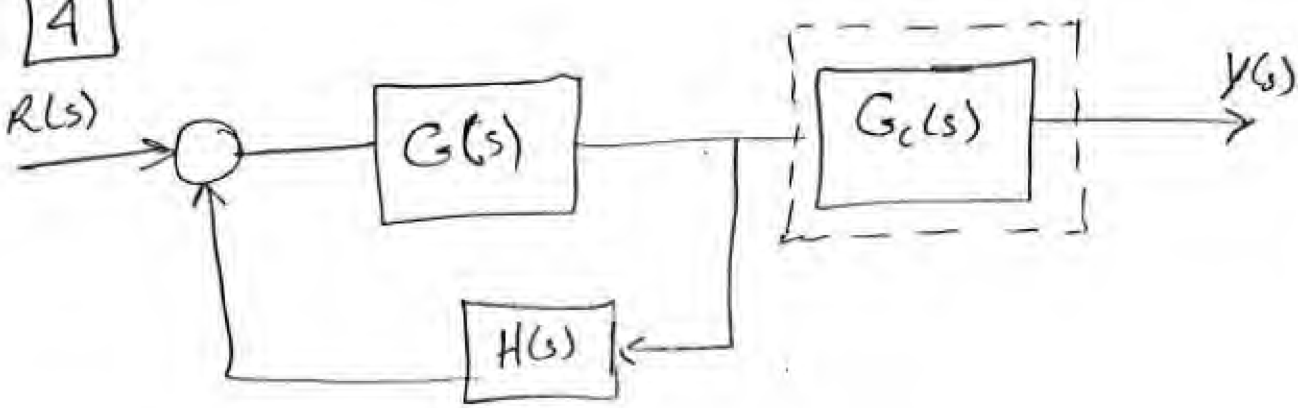

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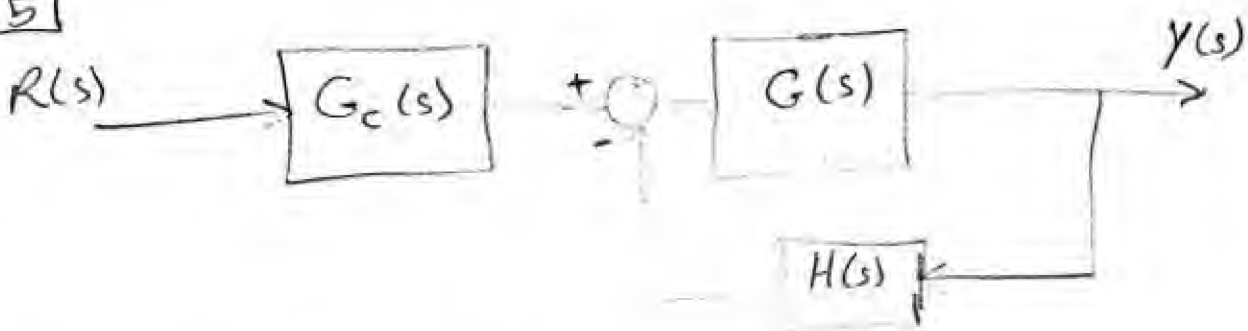


structure ① is the most used one.

Controllers → software controllers.
→ H/w controllers.

* Classification of Contr. 5

1 classical Controllers / traditional Controllers

* PI Controller. ⇒ (improve steady state error)

* PD Controller (improve system dynamics "speed up sys. response and reduce overshoot"
transient يقلل فترة ال 5

* PID (balance between PD & PI)

* Phase-lead Controller (the same as PD controller)

* Phase-lag Controller (The same as PI)

* Phase-lead-lag " (The same as PID)

[2] Modern Controllers

* Pole placement design/state-feedback control design

* state estimator /observer design

based on the states of the system.

$$\boxed{\begin{aligned} x &= Ax + Bu \\ y &= Cx \end{aligned}}$$

3 Computational (Inteligent Controllers)

→ the Controllers are built with the help of

(1) Fuzzy Control (FC)

(2) Neural network (NN)

(3) ~~Ada~~ Adaptive Neuro fuzzy Inference system (ANFIS) "Neural + fuzzy"

(4) Evolutionary Computation

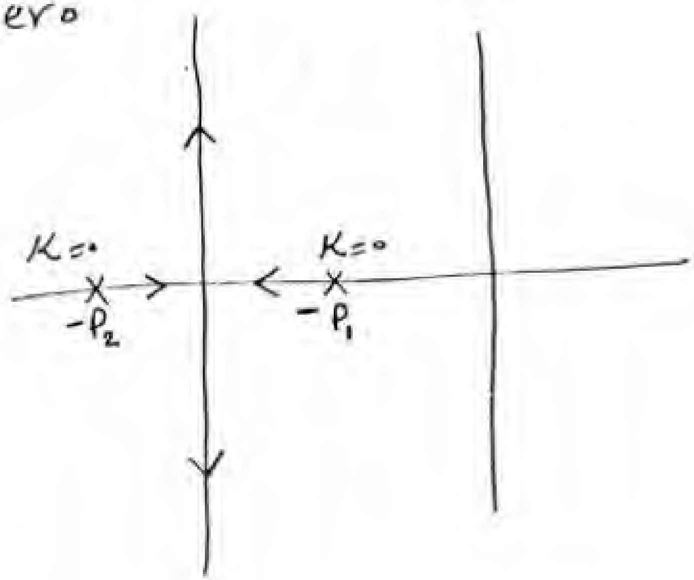
- → Differential evolution (DE)
- → Partical swarm optimization (PSO)
- → Genatic Algorithm (GA)
- → Artificial Bee Conlony Algorithm (ABC)

Phase-lead Controller

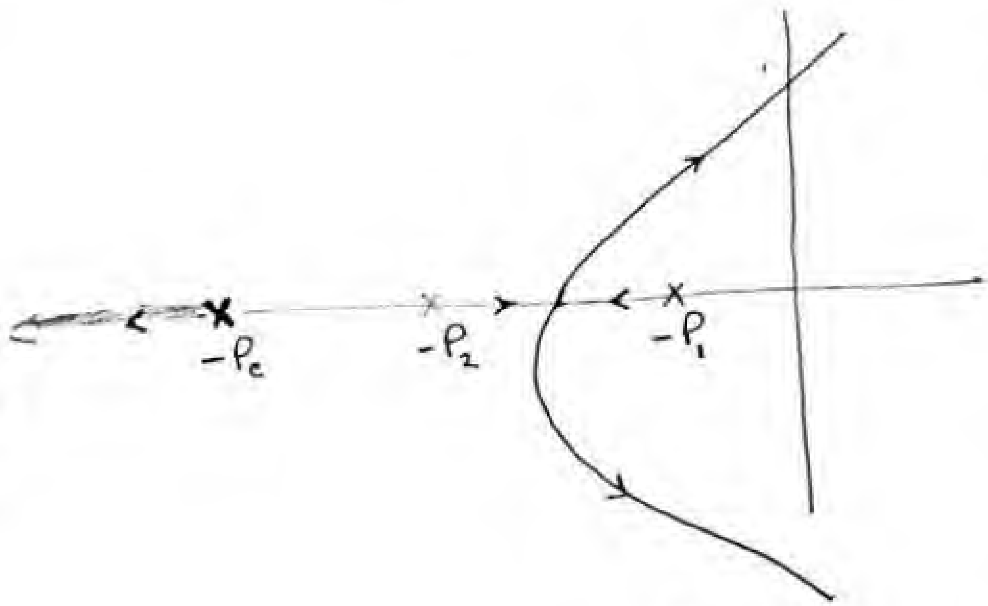
→ Effect of adding Poles / zero

$\boxed{Ex}$

$$GH(s) = \frac{K}{(s+P_1)(s+P_2)}$$

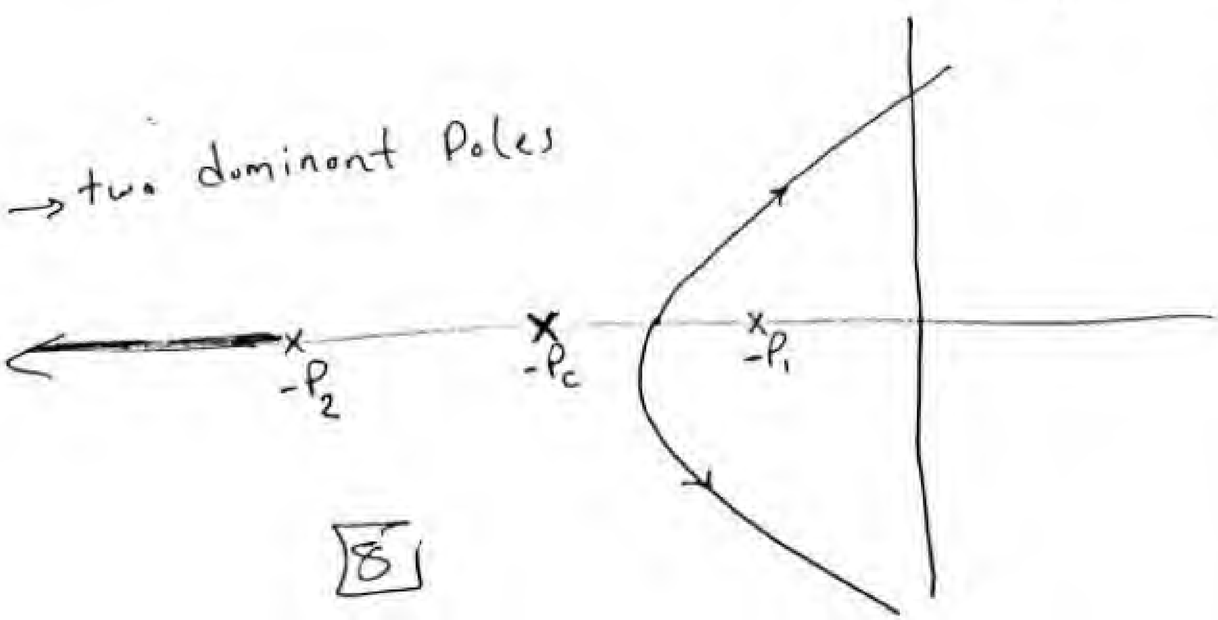


P_1, P_2 → two dominant Poles.



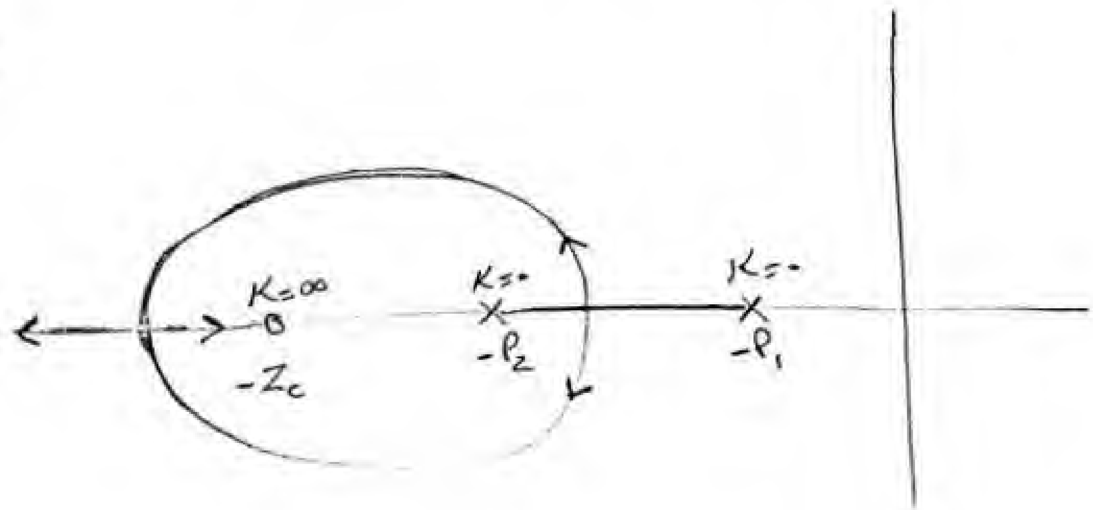
$-P_1, -P_2$ → dominant Pole

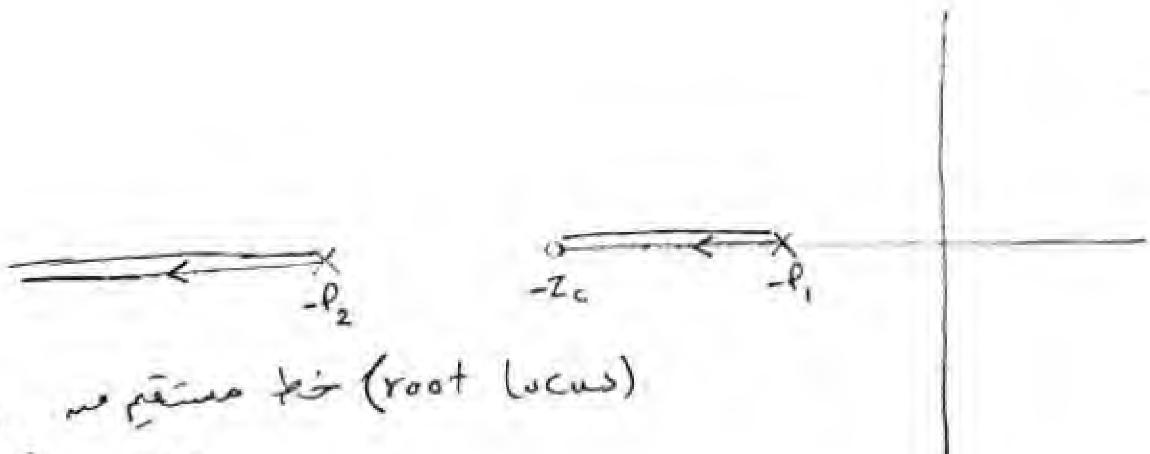
$-P_1, -P_c$ → two dominant Poles



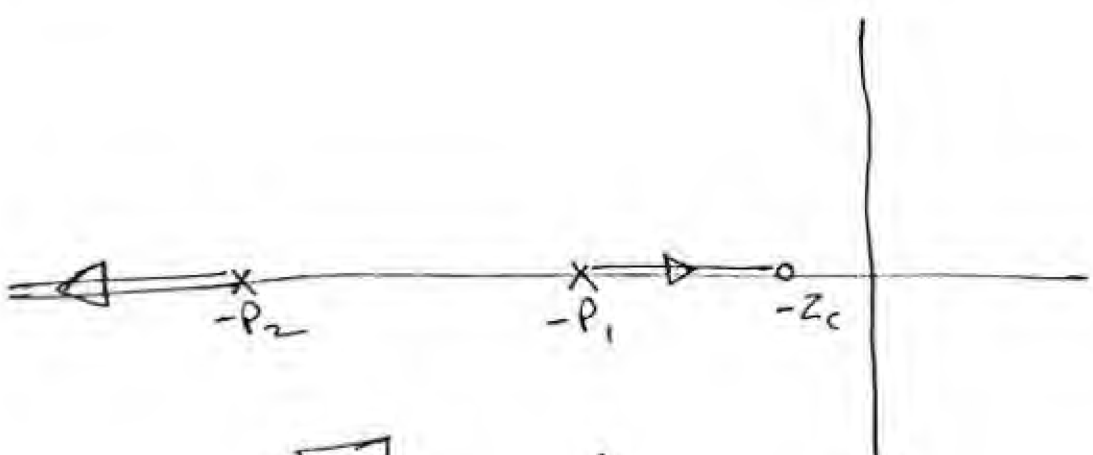
ـــ لاحظ اختلاف مكان الـ (Poles) غير في الـ (dominant Poles)
ما يغير في خواص الـ (root locus)

~> add zero



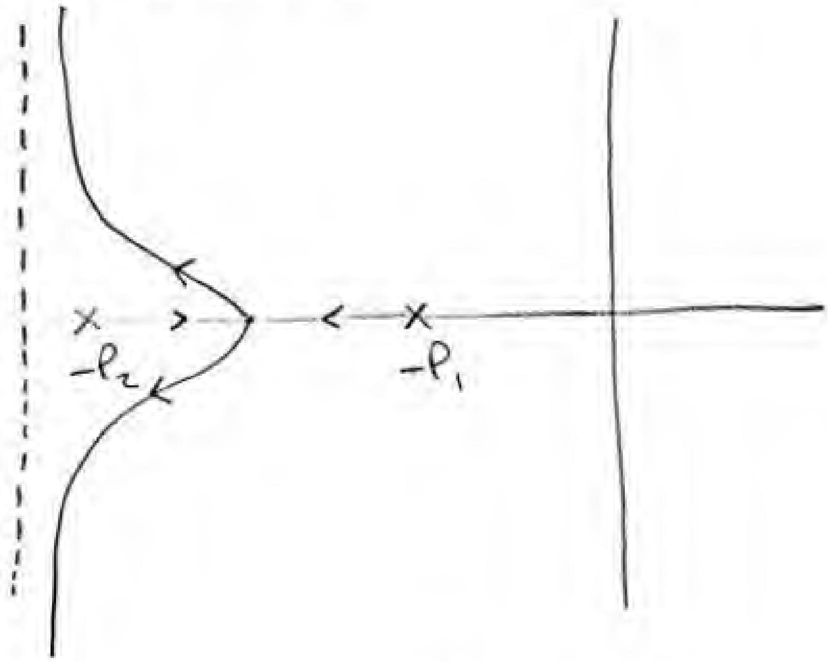


(root locus) خط مستقيم من $-P_1$ لـ $-Z_c$
ومن $-P_2 \leftarrow -\infty$



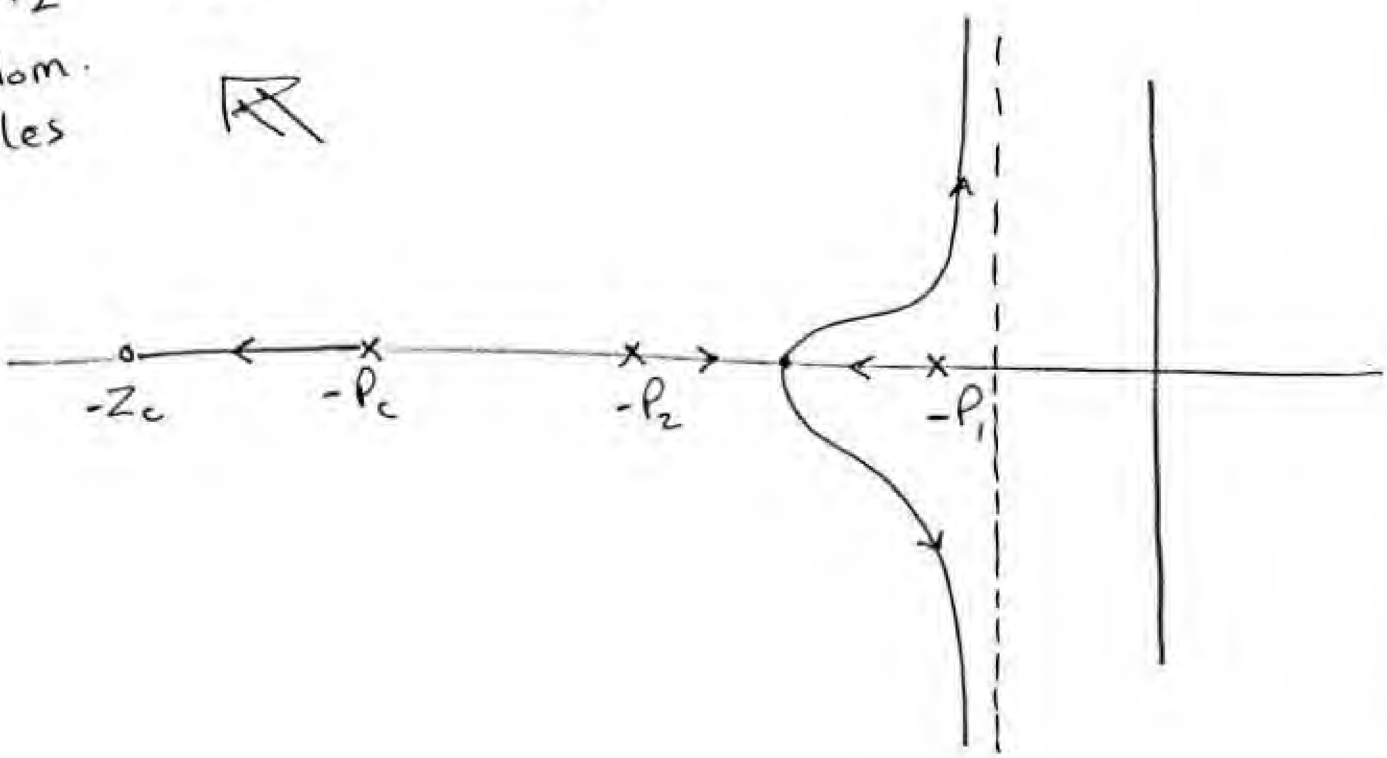
add Pole and zero at the same time

$$GH(s) = \frac{K(s+Z_c)}{(s+P_1)(s+P_2)(s+P_c)}$$

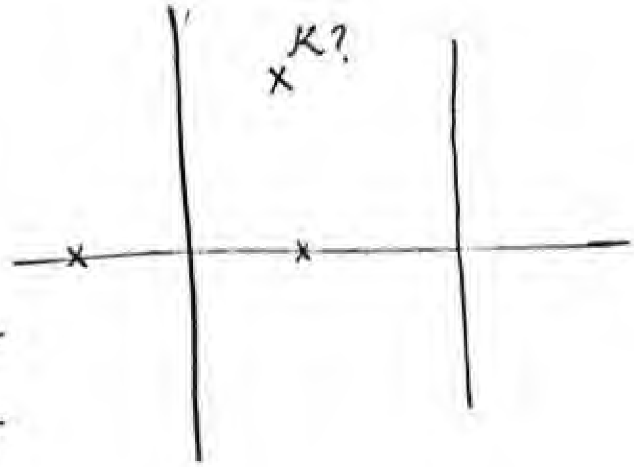


$-P_1, -P_2$

Two dom. poles



لو عايز K عند النقطة دى
فهى لا تنتمى للـ (root locus)



ـ لكن عندما أضيف Pole, Zero
جعلت من الممكن إن الشكل يمر
بالنقطة "x" .

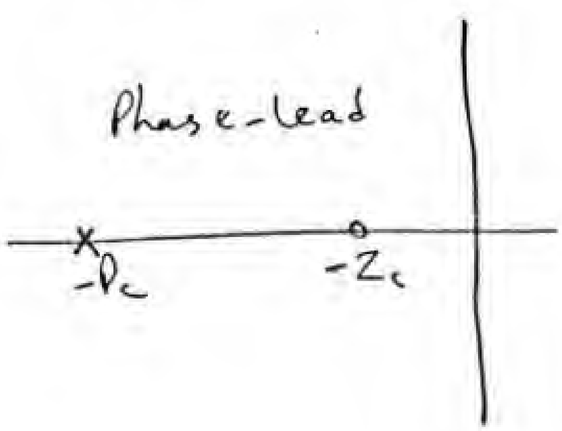
for Phase lead or lag Controller, Assume

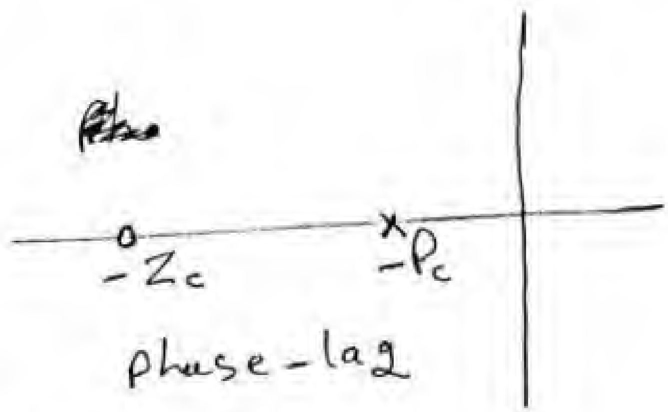
$$G_c(s) = \frac{s + Z_c}{s + P_c}$$

one zero at $-Z_c$
one Pole at $-P_c$

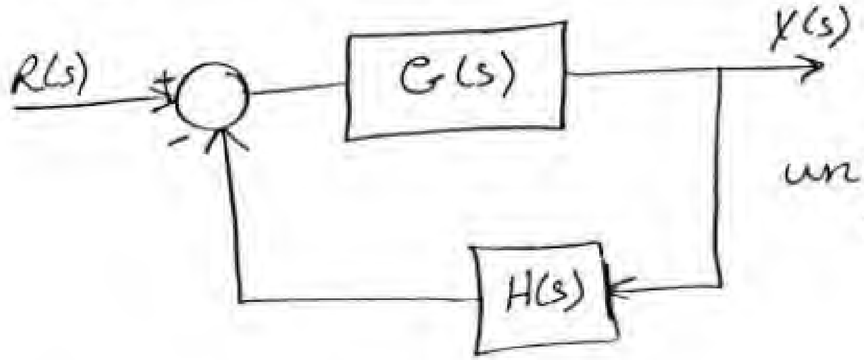
$|Z_c| < |P_c| \Rightarrow$ Phase lead Controller

$|P_c| < |Z_c| \Rightarrow$ Phase-lag Controller



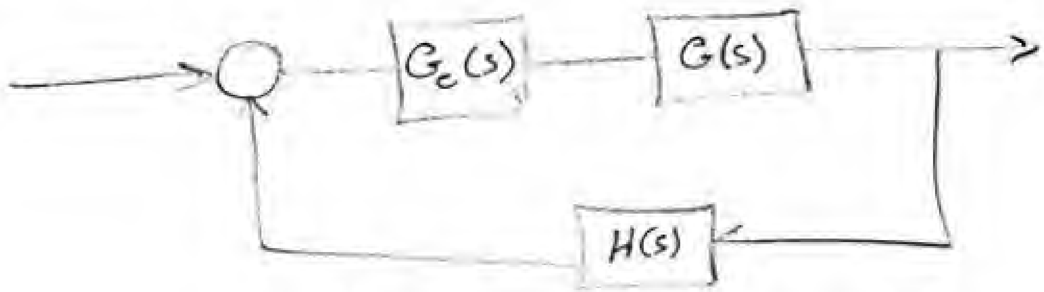


* The required is to find the location of Z_c, P_c that meet the required design spcs.



uncompansated sys.

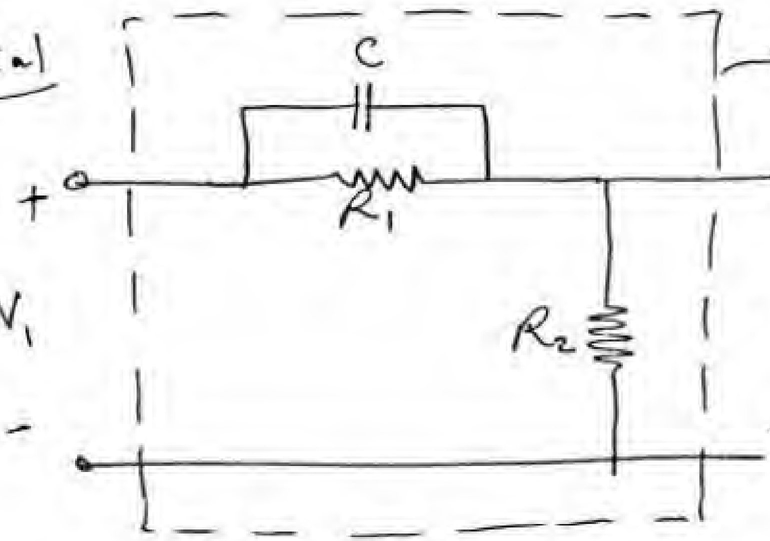
$$O.L.T.F = GH(s)$$



"compansated sys."

$$\begin{pmatrix} \text{Total} \\ O.L.T.F \end{pmatrix} = G_c(s) \cdot GH(s)$$

→ electrical



→ Controller

$$G_c(s) = \frac{s+Z_c}{s+P_c}$$

$$G_c(s) = \frac{V_2(s)}{V_1(s)} = \frac{I(s)\ R_2}{I(s)\cdot Z_{eq}}$$

$$Z_{eq} = \left(R_1 \,//\, \frac{1}{Cs}\right) + R_2 = \frac{R_1}{R_1Cs+1} + R_2$$

$$G_c(s) = \frac{R_2}{\frac{R_1}{R_1Cs+1} + R_2} = \frac{R_2(R_1Cs+1)}{R_1 + R_2(R_1Cs+1)}$$

$$= \frac{R_2R_1Cs + R_2}{R_1 + R_2R_1Cs + R_2} \qquad \div R_2R_1C$$

$$= \frac{s + \frac{1}{R_1C}}{s + \frac{R_1+R_2}{R_1R_2C}}$$

$$G_c(s) = \frac{s+Z_c}{s+P_c}$$

$$Z_c = \frac{1}{R_1 C}$$

$$P_c = \frac{R_1 + R_2}{R_1 R_2 C}$$

$$G_c(s) = \frac{s + Z_c}{s + P_c} = \frac{s + \frac{1}{\tau}}{s + \frac{1}{\alpha \tau}}$$

for prev. ex: $G_c(s) = \dfrac{s + \frac{1}{R_1 C}}{s + \frac{R_1 + R_2}{R_1 C R_2}}$

$$\tau = R_1 C \qquad , \quad \alpha = \frac{R_2}{R_1 + R_2}$$

$$G_c(s) = \frac{s + Z_c}{s + P_c} \qquad |Z_c| < P_c$$

The design steps to find Z_c, P_c

① using the given required specs

($\zeta, \omega_n, t_r, M_p, \dots \Rightarrow$ sys. dynamics)

To obtain the location of the desired poles ($s_{d_{1,2}} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$)

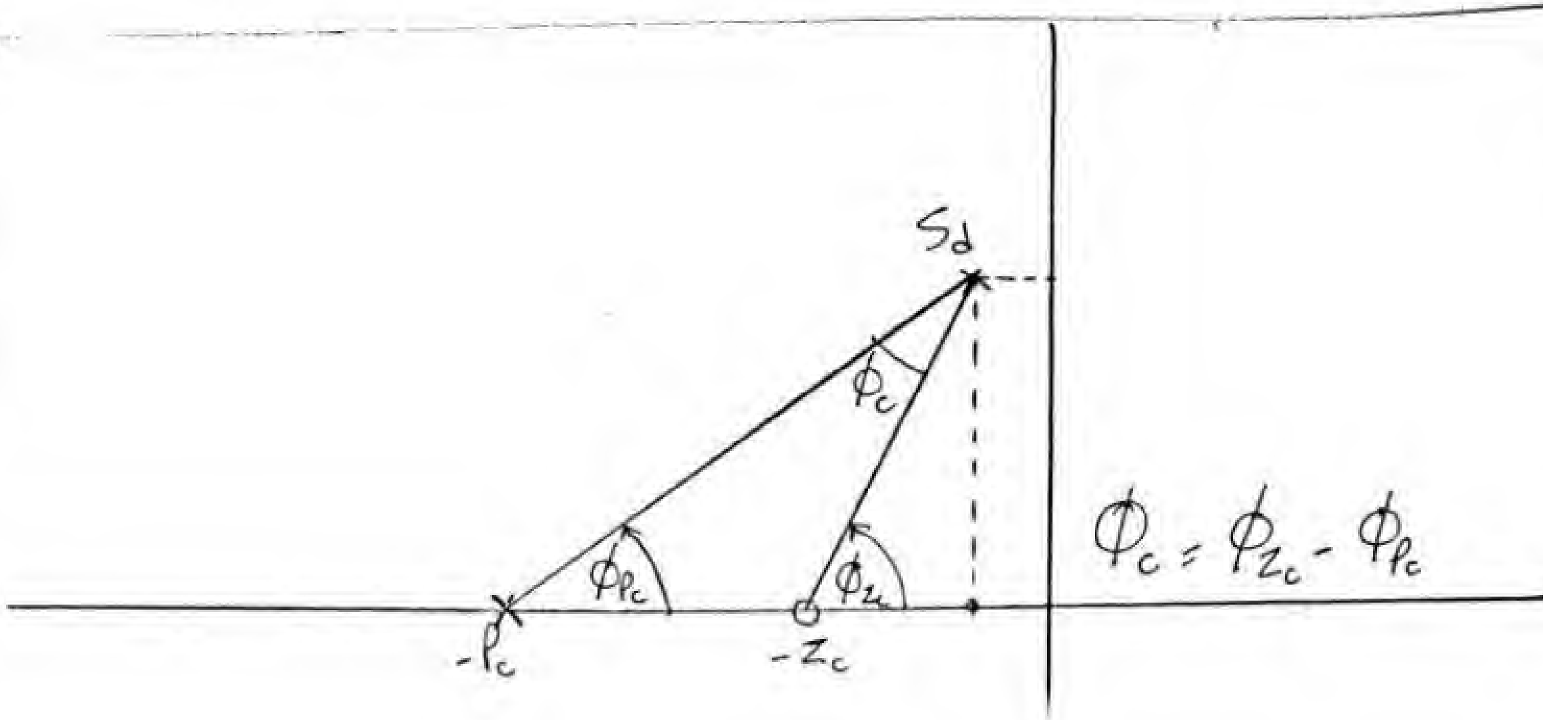
Ex: $s^2 + 2\zeta\omega_n s + \omega_n^2 \Rightarrow s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

② Apply the angle condition to obtain the compensator angle (ϕ_c):

$$\boxed{\angle GH + \phi_c = -180}$$

note that

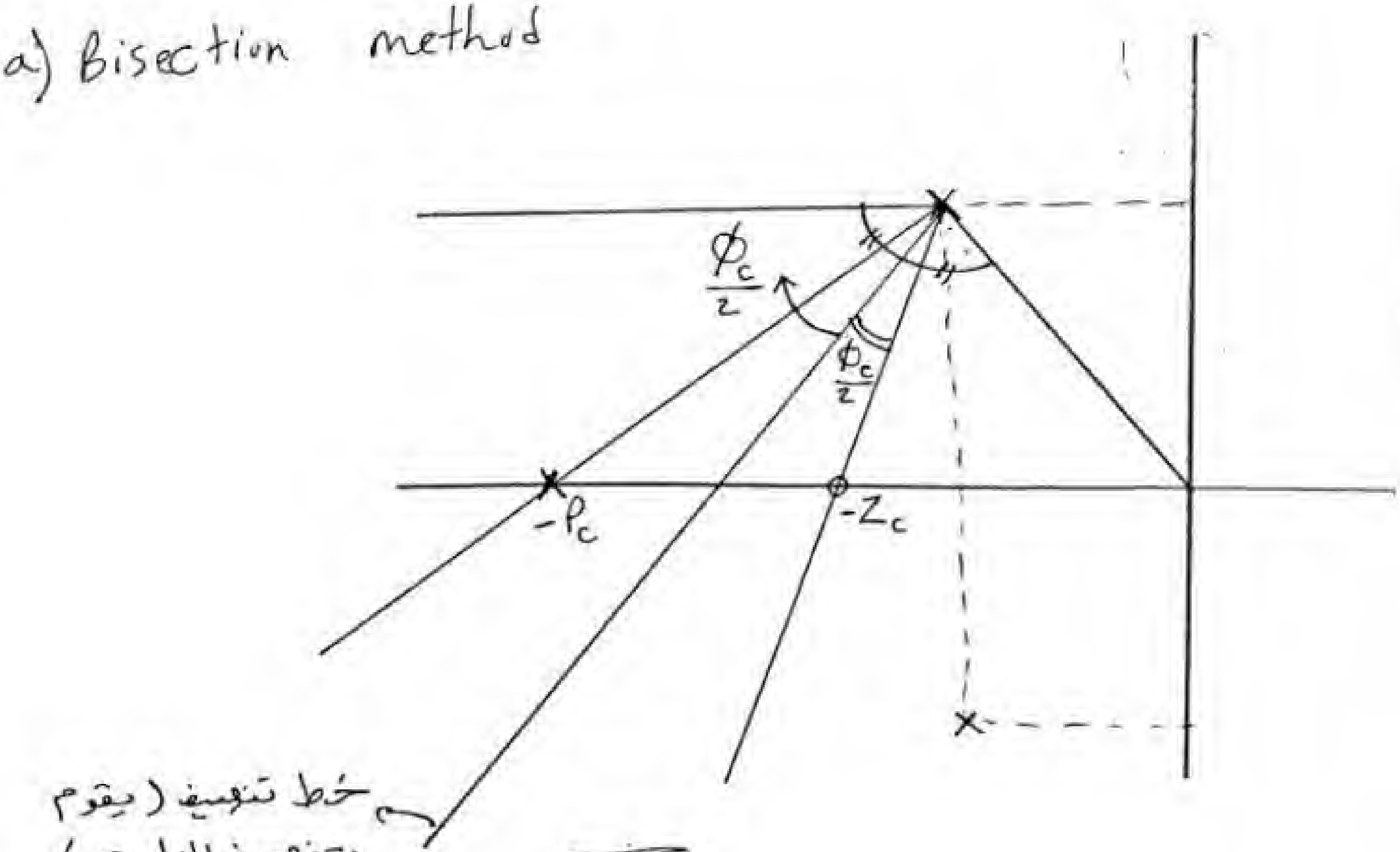
$$\phi_c = \phi_{Z_c} - \phi_{P_c}$$



$$\phi_c = \phi_{Z_c} - \phi_{P_c}$$

$\boxed{3}$ Determine the location of Z_c, P_c by the Known of ϕ_c

a) Bisection method



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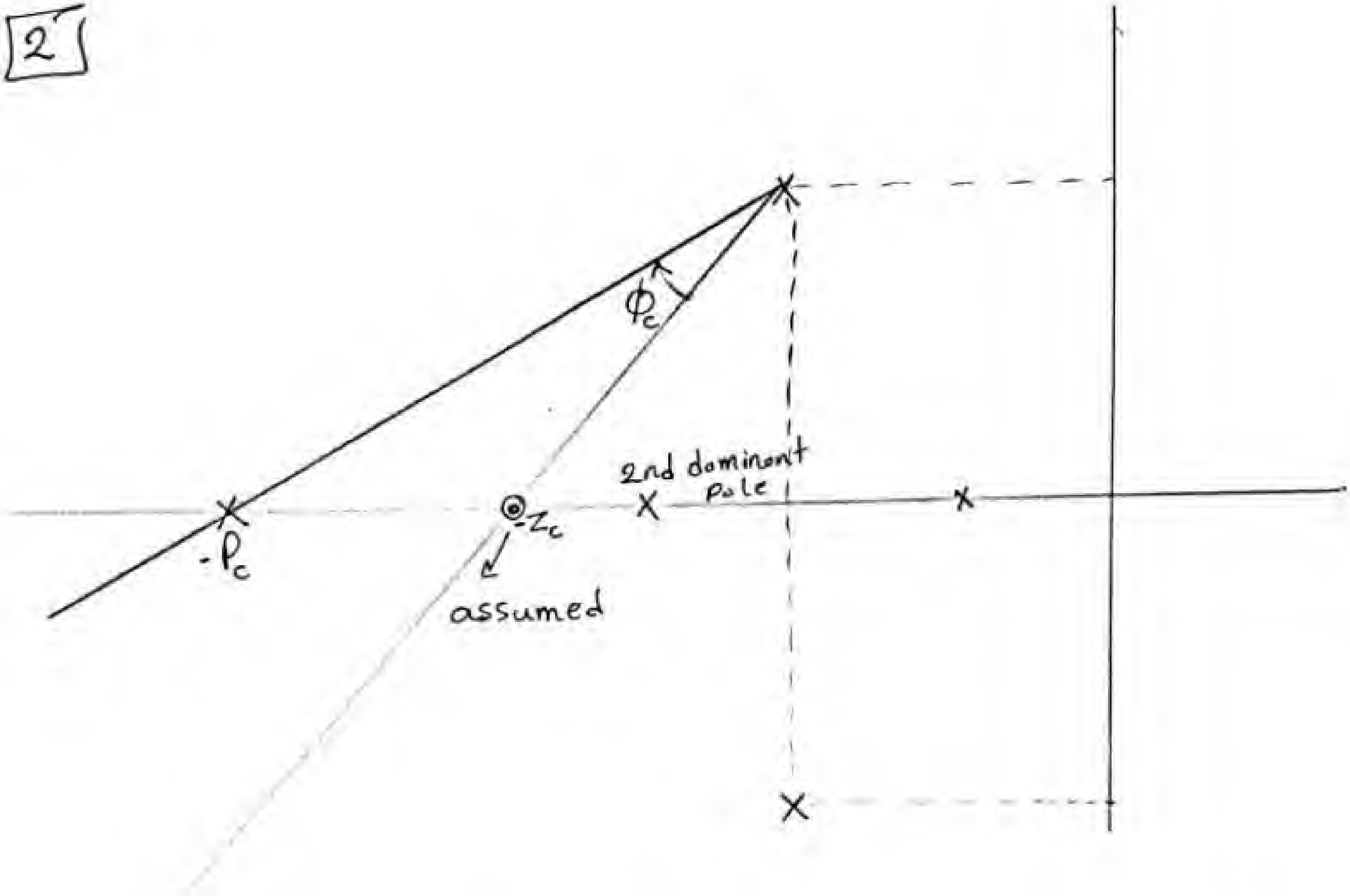
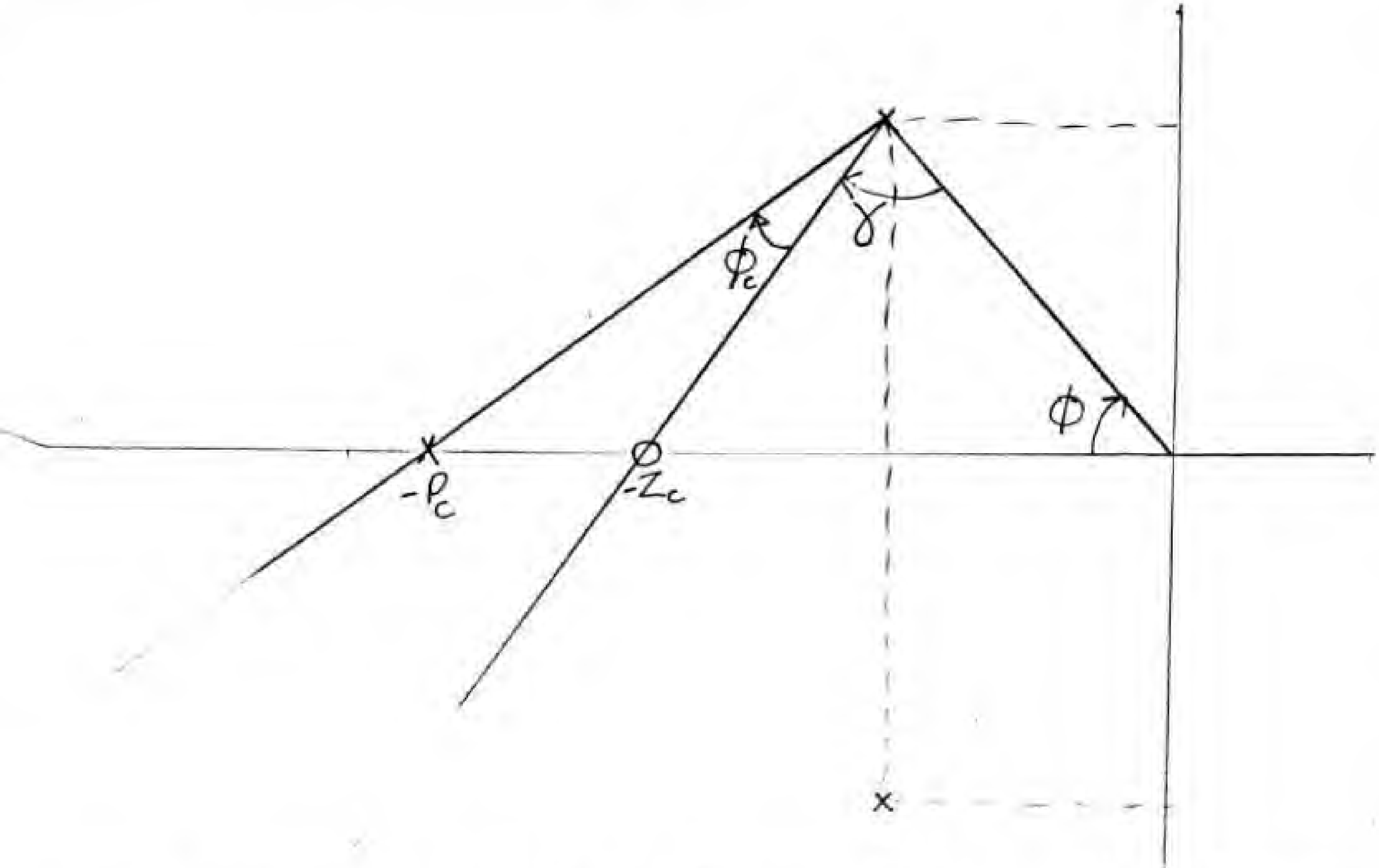


ـــ بيختوا ال (2-dominant Poles) ويفترض مكان ال (Zero)
على شمال ال (2nd Pole)

3 **Max. attenuated ratio**

$$\gamma = \frac{1}{2}\left[\pi - \phi - \phi_c\right]$$

$$\phi = \cos^{-1} \zeta$$



$\boxed{4}$ Apply the magnitude Condition To determine the value of gain "K" to meet desired specs:-

$$\left\| K\, G_c(s)\, GH(s) \right\|_{s=s_d} = 1$$

s_d ↳ desired

or

$$K = \frac{\prod \text{Poles}}{\prod \text{Zeros}}$$

$\boxed{Ex}$ $$GH(s) = \frac{K}{s(s+1)(s+4)}$$

Design a Compensator to meet the following specs: $\zeta = 0.5$ & $\omega_n = 2$ rad/sec.

What is the type of Compensator:-

as the desired specs are concerned with system dynamics $\Rightarrow$ The Controller is lead controller

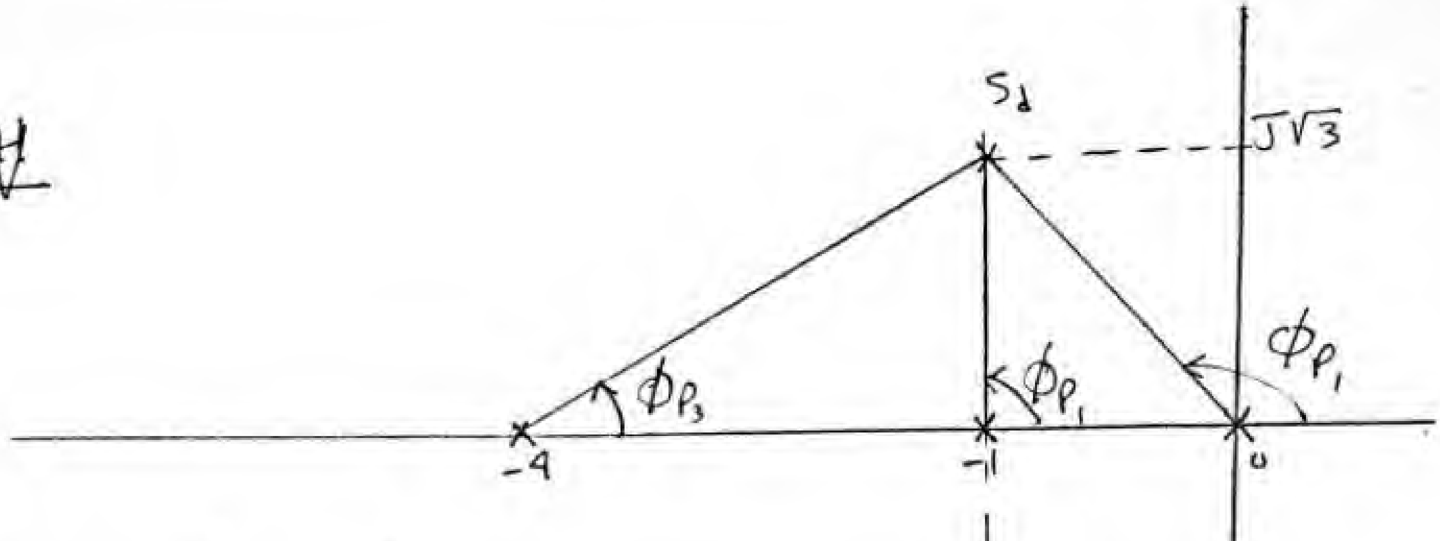
$\boxed{1}$ $S_{d_{1,2}} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ Location of desired Poles

$$= -1 \pm j\sqrt{3}$$
$\sqrt{3} \to 1.73$

$\boxed{2}$ Apply the angle Condition to get

$$\phi_c = \phi_{zc} - \phi_{Pc}$$

$\angle GH$



$$\angle GH = \underbrace{\phi_z}_{\text{مجموع زوايا ال zero}} - \underbrace{\phi_{p_1}}_{\text{مجموع زوايا ال pole}}$$

$$-\phi_{p_1} - \phi_{p_2} - \phi_{p_3} = \angle GH$$

$$\angle GH + \phi_c = -180$$

$$-\left[180 - \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)\right] - 90 - \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) + \phi_c = -180$$

$$\boxed{\phi_c = +60^\circ}$$

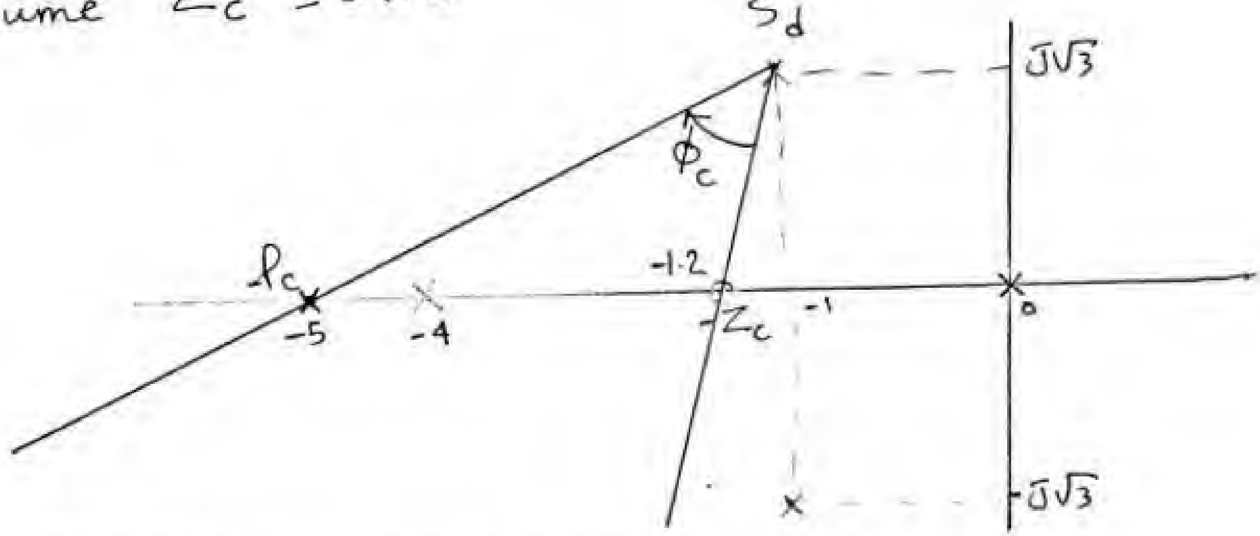
3 Determine the location of Z_c, P_c

~~assume~~

$-1, 0 \rightarrow$ two dominant poles

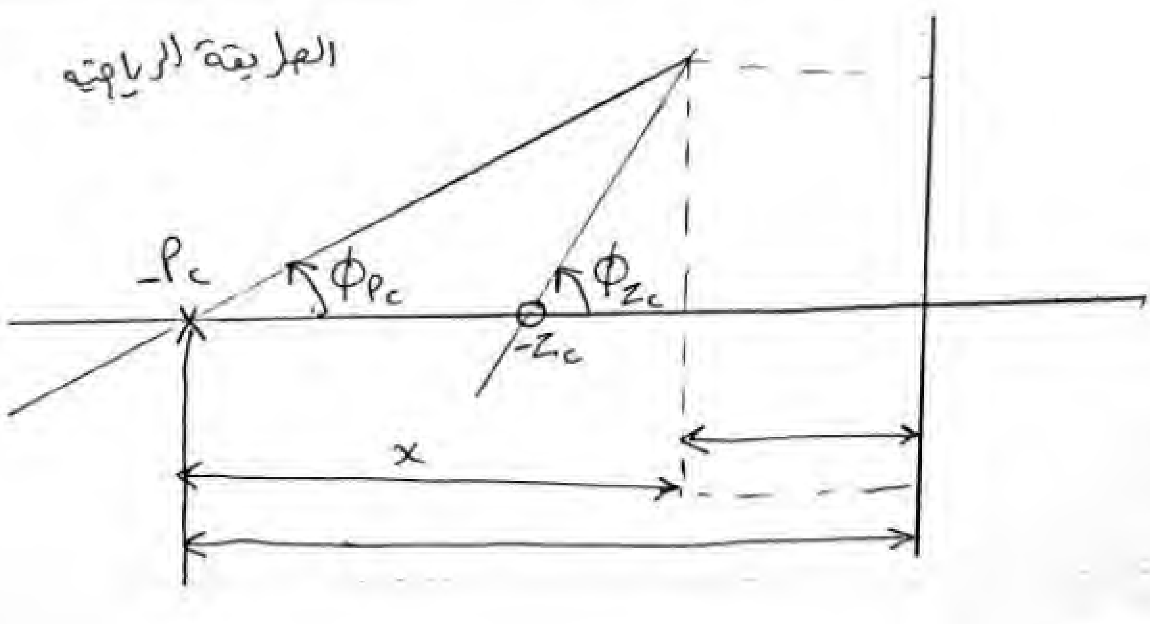
~~assume~~ locate Z_c on the left of the 2nd dominant pole (-1)

assume $Z_c = -1.2$



$-P_c = -5$, $-Z_c = -1.2$

الطريقة الرياضية



$$|P_c| = 1 + x$$

$$\tan(\phi_{P_c}) = \frac{\sqrt{3}}{x}$$

$$\phi_{Z_c} = \tan^{-1}\left(\frac{\sqrt{3}}{0.2}\right) = 83.413^\circ$$

$$\phi_c = \phi_{Z_c} - \phi_{P_c}$$

$$60 = 83.413 - \phi_{P_c} \Rightarrow \phi_{P_c} = 23.41^\circ$$

$$\tan(\phi_{P_c}) = \frac{\sqrt{3}}{x} \Rightarrow x = 4$$

$$|P_c| = 1 + 4 = 5$$

Total O.L.T.F $= K \cdot G_c(s) \cdot GH(s)$

$$= \frac{K(s + Z_c)}{s(s + P_c)(s+1)(s+4)}$$

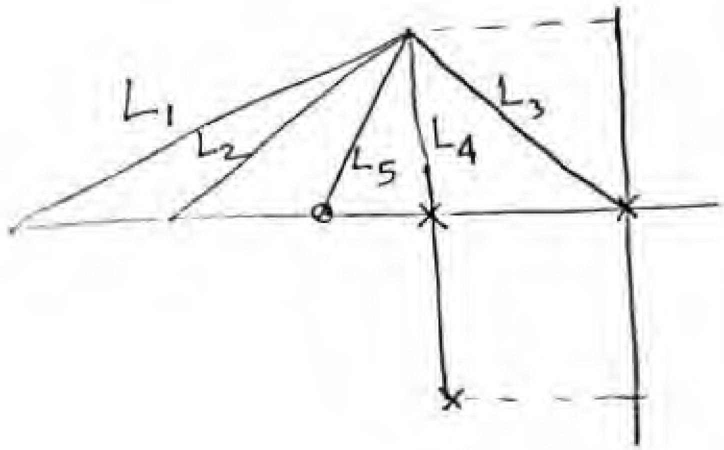
$$\boxed{= \frac{K(s + 1.2)}{s(s+5)(s+1)(s+4)}}$$

$\boxed{4}$ K at $s_{d_{1,2}} = -1 \pm j\sqrt{3}$

$$\left\| K \cdot G_c(s) \cdot GH(s) \right\|_{s=s_d} = 1$$

$$K = \left\| \frac{s(s+5)(s+1)(s+4)}{(s+1.2)} \right\|_{s=-1+j\sqrt{3}}$$

$$\boxed{K = 30}$$



or $K = \dfrac{\Pi \text{ Poles}}{\Pi \text{ zeros}}$

$$= \frac{L_1 L_2 L_3 L_4}{L_5}$$

$\boxed{5}$ check steady-state error (e_{ss})

$r(t) = 1 \Rightarrow K_p = \lim_{s \to 0} GH(s)$

$\hookrightarrow$ s.s.e $= e_{ss} = \dfrac{1}{1+K_p}$

$$r(t) = t \Rightarrow K_v = \lim_{s \to 0} s\, GH(s)$$

$$\Rightarrow e_{ss} = \frac{1}{K_v}$$

$$r(t) = \frac{t}{2} \Rightarrow K_a = \lim_{s \to 0} s^2\, GH(s)$$

$$\hookrightarrow e_{ss} = \frac{1}{K_a}$$

$$K_v = \lim_{s \to 0} s . K . G_c(s) . GH(s)$$

$$= \lim_{s \to 0} s \frac{K(s+1.2)}{s(s+1)(s+4)(s+5)}$$

$$= \frac{30\,(1.2)}{(1)(4)(5)} = 1.8$$

$$s.s.e = \frac{1}{K_v} = \frac{1}{1.8} \simeq 0.56$$